THE TESTING OF TRANSFORMER STEEL.

By M. G. Lloyd and J. V. S. Fisher.

Many methods have been employed for the testing of sheet iron and steel for energy losses when subjected to alternating magnetization, but all that have so far been employed have been lacking in some desirable qualifications. The plotting of a hysteresis loop from readings of magnetizing force and the resulting magnetic induction, and the measurement of the area of this loop, is a slow and tedious process and gives no indication of the eddy current losses. Methods depending upon the relative motion of the specimen and a magnet, necessitate an air-gap in the magnetic circuit, with a resulting induction in the specimen which is far from uniform. Moreover, if a permanent magnet be used, as in the Ewing and the Blondel apparatus, one is restricted in the values of the flux density which may be used, and the apparatus is only suitable for comparative measurements at one flux density.

Consequently, methods of testing with alternating currents have come to be regarded as the only satisfactory way of making these measurements, and present efforts are directed to securing the best conditions for this form of test. In the wattmeter method, the electrical energy which is supplied to maintain the alternating magnetization is measured with a wattmeter, while the maximum magnetic induction produced in the specimen is determined by a voltage measurement at the terminals of the magnetizing winding or at the terminals of a secondary winding placed around the same core of test material.

The specimen may be employed in three forms. (1) It may be in the form of straight strips placed in contact with a yoke, thus forming a closed circuit of ferromagnetic material. (2) The straight strips may be used without any yoke. (3) The specimen may be arranged to form a closed magnetic circuit in itself.

The second form gives a distribution of flux which is far from uniform, and is therefore objectionable. The first form gives a more uniform flux, but it is necessary to distinguish between the energy supplied to the specimen and that supplied to the yoke. This can only be done satisfactorily by knowing the constants of the yoke, and only then by having the distribution of flux uniform, a condition difficult to secure. Consequently, for accurate measurements the third form is the most reliable, although for factory use the first or second may prove more convenient where accuracy can be sacrificed for other considerations.

Assuming then a closed magnetic circuit of the material, to be tested by the wattmeter method, the following conditions should be realized as far as possible.

1. The flux should be uniformly distributed over the cross-section of the specimen, and should be the same at every section. This requires that there should be no leakage of flux through the air.

2. A definite form of wave of magnetic flux should be used, or in other words, a definite form of wave of secondary electromotive force, since the form factor of this wave enters into the computation of maximum flux density from the observed effective voltage.

3. The material used should be cut in a form such that only a small part of it is contiguous to a cut edge, since it is well known that all methods of cutting have a hardening effect upon the material bordering upon the cut. This means that the strip, whether straight or in ring form, should not be too narrow. This condition may be dispensed with if all specimens are annealed under definite conditions after cutting to size, and prior to testing.

4. The amount of material required should not be greater than is necessary to get a fair average value.

5. The corrections to be made to the readings of the instruments to get final values should be as few and as small as possible.

Two general forms of magnetic circuit are available. The material may be stamped into rings, or the circuit may be built up from straight strips. Leakage is most effectually avoided by using rings. With this form of specimen, however, it is impossible to satisfy simultaneously conditions 1 and 3, unless rings of very great diameter are employed, and in the latter case there is a very great waste of material. The nonuniformity of flux existing in rings of small diameter, even when uniformly wound, and the errors resulting therefrom, have been discussed in a previous article.[1] The use of rings is thus restricted to cases where the material is annealed after stamping, and the radial width of the ring should be very small in comparison to its diameter. When rings are employed, the labor of winding each specimen separately with a magnetizing coil may be obviated by the use of the apparatus of Esterline [2] or Möllinger.[3]

To meet condition 2 it is sufficient to know or measure the form factor of secondary voltage.[4] By making runs at two frequencies it is then possible to separate the eddy current and hysteresis losses, and, if desired, to compute the eddy current loss for a standard wave form.[5]

It is far preferable, however, to work throughout with a sine wave when a generator is available which will fulfill this condition. There are three things which may prevent the realization of this condition. In the first place the machine may not generate a sinusoidal electromotive force. In fact, it may be stated as a general proposition that no generator gives a perfect sine wave. The only question is as to the magnitude of the harmonics present, and whether these are negligible. It can not be assumed that these are negligible simply because the machine was designed to give a sine wave, or because a rough oscillogram does not indicate definite distortion. The only way to be certain is to take an accurate curve from the machine and analyze it by measurement of the ordinates.

As an example of the necessity of this, we cite an instance occurring at the Bureau of Standards. Here the tests are usually made with a generator whose emf. wave contains a third harmonic

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[1] M. G. Lloyd, this Bulletin, **5**, p. 435; reprint No. 108.

[2] J. W. Esterline, Proc. Am. Soc. Testing Materials, **3**, p. 288; 1903.

[3] J. A. Möllinger, Elektrot. Zs. **22**, p. 379; 1901.

[4] An apparatus for measuring form factor is described by Lloyd and Fisher this Bulletin, **4**, p. 469; 1908. Reprint No. 87.

[5] See M. G. Lloyd, this Bulletin, **5**, p. 381; 1909. Reprint No. 106.

whose amplitude is 0.6 per cent of the amplitude of the fundamental, and none of the higher harmonics are present to such an extent as 2 per cent. The form factor is almost exactly that for a sine wave. One day a specimen, which had already been tested with this generator, was tested with a second generator supposed to give a sine wave, and whose oscillogram appeared smooth and inoffensive. The losses appeared more than 4 per cent lower than by the previous test. This led to a closer examination of the wave given by the generator, which was traced by the Rosa apparatus,[6] analyzed and found to contain nearly 7 per cent of the third harmonic, sufficient to account for the observed difference in losses.[7]

A second cause of distorted wave form is to be found in armature reaction, which may alter the emf. of a loaded generator when the curve on no load is sinusoidal. For this reason, it is best in testing to use a machine so large that it is only slightly loaded by the test current.

A third cause of distorted wave is to be found in the drop of potential due to the ohmic resistance of the circuit. The generator emf. is made up of two parts, one of which is balanced by the emf. induced by the changing flux, while the other produces current. If the flux be sinusoidal, the emf. induced by it is also sinusoidal. But owing to the fact that the permeability of the material varies with the magnetic induction, the magnetizing current can not be sinusoidal. The component of emf. producing current has the same form of wave as the current, and it also can not be sinusoidal. The total emf. of the generator, then, to produce sinusoidal flux, is made up of one component which is sinusoidal and one which is not, and therefore is not sinusoidal. Its shape must vary with the conditions, and hence it would be useless to attempt to secure such a form of generator emf. To approximate sinusoidal flux it is necessary to have a sinusoidal emf. at the generator, and to make the component of this, which sends current (equal to the product of current and resistance) negligible in comparison with the component which balances the emf. induced in the apparatus.

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[6] See Rosa and Grover, this Bulletin, **1**, p. 138; 1905. Reprint No. 9.

[7] See M. G. Lloyd, this Bulletin, **4**, p. 484; 1908. Reprint No. 88

It is desirable then to keep both resistance and current low in the magnetizing circuit. If this ohmic drop of potential can be made negligible, then the wave form of flux will differ from a sine curve only by a negligible amount.

The resistance in the magnetizing circuit consists of the armature, leads, magnetizing coil, measuring instruments and perhaps of windings of transformers used to step up or down to the proper voltage. It should not include a regulating rheostat, but the current should be controlled through the generator field. Each of these items should be made as low as possible, and thus appears another reason for choosing a generator of capacity large in comparison to the load to be placed upon it. The magnetizing current may be kept low by having a magnetic circuit of low reluctance. Air gaps should be avoided, and any joints in the magnetic circuit made as good as possible.

With the same magnetizing current, the induced emf. is proportional to the cross-section of test material; consequently the greater the quantity of material used, the less the distortion of wave. With a definite cross-section and windings, the induced emf. is proportional to the maximum flux density, but the magnetizing current is not proportional to the flux density, owing to varying permeability. The larger the permeability, the larger the ratio of flux to magnetizing current. The distortion will consequently be less if the iron be magnetized in the region of maximum permeability, and the distortion is sure to become appreciable if the flux density be carried too high, and may become appreciable at very low flux densities, even when it is negligible through the range of working flux densities used industrially.

In the method of Epstein,[8] which has been adopted as standard in Germany,[9] these conditions are fairly well met. Ten kilograms of sheet are cut into strips 50 by 3 cm and assembled into four bundles, over which solenoids are slipped. The four bundles are arranged in the form of a square, having butt joints at the corners, where the magnetic material is separated by a sheet of

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[8] I. Epstein, Elektrot. Zs. **21**, p. 303; 1900.

[9] Elektrot. Zs. **24**, pp. 657, 684; 1903.

thick paper. This interruption to the magnetic circuit tends to make the flux more uniform across the section of test material, but it also makes the leakage greater, and the flux less equal at different sections. Thus the flux at the center of one bundle may exceed the flux near one end by as much as 8 per cent. This air-gap also makes the reluctance of the magnetic circuit high, and consequently the magnetizing current high, and a large quantity of test material must be used. Condition 4 is here sacrificed for the better attainment of conditions 1, 2, and 3, and yet conditions 1 and 3 are not very well satisfied.

A modification of this method has been developed at the Bureau of Standards which differs from the above principally in the arrangement of the test material. A smaller quantity of material in wider strips may be used, while at the same time a greater uniformity of flux is secured. The amount of material used is from 1.5 to 2 kg (less than 4 pounds), and an accuracy of 1 per cent is attained.

DESCRIPTION OF THE APPARATUS.

The specimen to be tested is cut into strips 25.4 by 5 cm (10 by 2 inches). These are assembled into four bundles, in each of which adjacent strips are separated by strips of press board of equal width and thickness, but 2 cm shorter. Each bundle is wrapped with friction tape, and is inserted in a solenoid, and the four are then arranged in a square so that the plan view shows the edges of the strips. (See Fig. 1.) The solenoids are wound upon fiber frames which are 22.7 cm long, and have inside dimensions 5 by 1 cm. At the corners of the square, short pieces of test material are bent at right angles and interleaved between the strips of adjacent bundles, as shown in the figure. There are as many of these corner pieces as there are test pieces, and they are graduated in length so as to give a uniform lap of about 2 mm. A special clamp, shown in Fig. 2, is tightened over these laps, so as to give a good magnetic joint.

Each solenoid has in its first layer two windings of double-silk-covered No. 20 wire, each consisting of 45 turns. Over these are wound 250 turns of No. 14 copper wire, also double-silk-covered, to form a magnetizing coil. The four solenoids are connected in series, making a total of 1000 magnetizing turns and two secondaries of 180 turns each. One of these secondaries is connected to a voltmeter for determining the magnetic flux. This instrument is a deflecting mirror dynamometer, giving a sufficient deflection with 0.004 ampere. The other secondary is connected

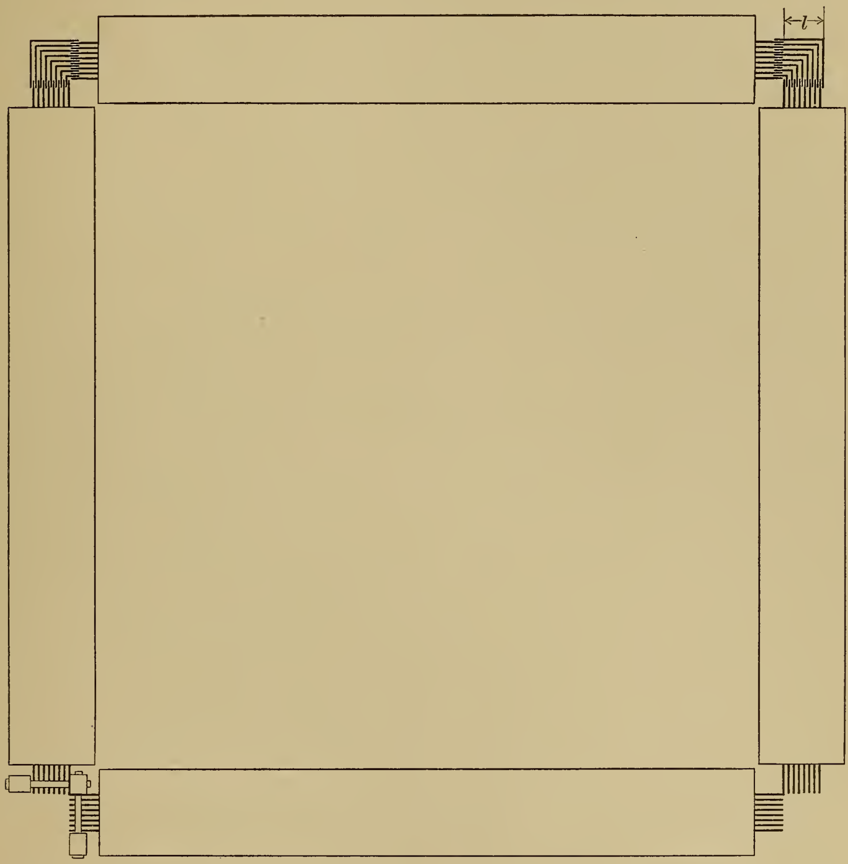

Fig. 1.—Plan of Apparatus with Test Pieces in Position. Corner pieces have been removed from two and clamps from three corners.

to the moving coil circuit of a watt dynamometer of the same type as the voltmeter. The magnetizing current traverses the field coils of this wattmeter, whose deflections are a measure of the power supplied to the core and the secondary coils. The copper loss in the primary is thus eliminated from the power measurement,[10] as is evident from the following considerations.

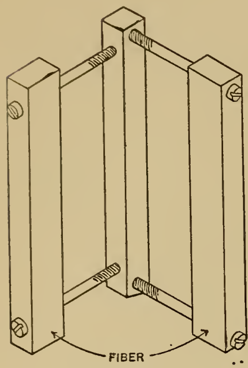

Fig. 2.

Let

N_1 = primary turns.

N_2 = secondary turns.

Φ = flux threading both primary and secondary.

i = primary or magnetizing current.

e = emf. applied to primary.

L = self-inductance of primary due to any flux not included in Φ.

Then

$$e - N_1 \frac{d\Phi}{dt} - L \frac{di}{dt} = ri$$

The instantaneous power expended is

$$ei = ri^2 + N_1 i \frac{d\Phi}{dt} + Li \frac{di}{dt}$$

The integral of this expression, extended over a complete cycle, will give the net power. The term ri^2 represents the primary copper loss. The term $Li \frac{di}{dt}$ when integrated over a complete cycle, is equal to zero. The term $N_1 i \frac{d\Phi}{dt}$ will not integrate to zero when there is either hysteresis in the core or secondary (including eddy) currents flowing, since in either case Φ is not in phase with i. This term represents the power expended in the core and in the secondary circuits. $N_2 i \frac{d\Phi}{dt}$ is proportional to this, and its integral value represents the reading of the wattmeter when connected as in this apparatus. So that the wattmeter reading, when multiplied by $\frac{N_1}{N_2}$, gives the power expended in the core and in the secondary circuits. Error will arise only when there is flux threading the core and linked with the primary, which is not linked with

[10] Due to C. P. Steinmetz, Trans. A. I. E. E. 9, p. 624; 1892.

the secondary. This is avoided by winding the secondary under the primary, and making the two coextensive in length. The energy in each secondary is obtained by squaring the secondary voltage and dividing by the resistance of its circuit. By using a low number of turns in the secondaries and sensitive instruments, these corrections are kept very small and are accurately known.

Voltmeter and wattmeter each have variable multipliers, whose resistance is adjusted to give a suitable deflection in each case. The accuracy of reading is usually better than 0.1 per cent, and is higher than the conditions require.

The frequency is determined by a Hartmann and Braun frequency meter, which has been calibrated by the use of a chronograph, or where greatest accuracy is required the chronograph is used directly.[11]

When it is desired to measure the magnetizing current, an ammeter can be introduced into this circuit, but the magnetizing current for ordinary inductions is so low that it is difficult to secure an ammeter of sufficiently low resistance. The only type of portable instrument which has answered this purpose is the Duddell thermo-ammeter. This can be obtained with a range of 0.5 ampere and a resistance of 0.2 ohm.

The use of corner pieces bent at right angles caused at first some apprehension as to its effect upon the results. It is known that bending, like any other mechanical treatment, will change the magnetic properties of the iron. If this affects enough of the iron to seriously alter the average value, it would condemn the method. Experiments which were directed to the determination of this effect showed that it is of no importance. The corner pieces are bent sharply in a machine which distorts the material to only a very short distance from the angle. The distorted material has its constants considerably changed without doubt, but as the corner pieces constitute only about 5 per cent of the entire circuit, and as only a part, say 30 per cent, of this material is altered by a fraction, say 20 per cent, of its initial value, an error of much less than 1 per cent is made by regarding this material as unaffected by the bending, and 1 per cent is the limit of accuracy claimed for the method.

[11] This use of the chronograph, with specimen record, is described by M. G. Lloyd, this Bulletin, 5, p. 388; 1909. Reprint No. 106.

To examine experimentally the effect of bending, a measurement was made in the usual way; the strips were then removed and each was bent at right angles and then bent straight at a point close to the first bend, so that the length was only slightly altered. A new measurement showed that the losses had increased by 1 per cent. A single bend would of course affect them by much less than this. Another experiment consisted in making measurements upon annealed and unannealed specimens, each with corner pieces of its own material. The unannealed corner pieces were then used with annealed test pieces, and after making the proper corrections and allowances for loss in the corner pieces (as hereinafter described) the values for the test pieces were found in agreement with those previously obtained with annealed corner pieces. As the annealed material is very much more sensitive to mechanical treatment than the unannealed, the effect must have caused different results in the two cases if it were operative to more than a negligible degree.

A third method of checking this point was tried, and led to the same conclusions, but the above are considered sufficient evidence.

On account of the lapping of the corner pieces over the ends of the test pieces, the flux density is low in this part of the material, and the results must be corrected therefor. The amount of lap is determined by the relative weights of corner and test pieces, as compared with the relative lengths of the two parts of the circuit. When the corner pieces are of the same material as the test pieces, it is assumed that the flux density is halved in the portion of material which laps, and the energy loss is consequently only one-third normal.

B = nominal (or average) maximum flux density.
B_1 = maximum flux density at ends of test pieces.
M = mass of test pieces.
m = mass of corner pieces.
l = dimension shown in Fig. 1.
W = measured loss.
$\frac{m}{M}$ = proportional increase in mass of magnetic circuit, due to corner pieces.

$\frac{l}{25.4}$ = proportional increase in length of magnetic circuit, due to corner pieces.

$\frac{m}{M} - \frac{l}{25.4} = c$ = mass of corner pieces which lap, expressed in terms of mass of test pieces.

$2c$ = total material (lapped and lapping iron) in which flux density is $\frac{B_1}{2}$

$2cW\left[1 - \left(\frac{B_1}{2B}\right)^x\right] = 2cWk$ = correction to W for lap, where x expresses the law of variation of loss with B.

k varies slightly with the conditions. If $B_1 = B$ (no leakage) and $x = 1.6$, $k = 0.67$. For $x = 2.0$ this becomes 0.70. For 4 per cent leakage, $x = 1.6$, $k = 0.68$. With sufficient accuracy for the purpose k may be taken as 0.70 throughout, so that the correction becomes 1.4 cW and the loss per unit mass is $\frac{W}{M+m}(1+1.4c)$ or $\frac{W}{(M+m)(1-1.4c)}$ with sufficient accuracy. The latter form is the most useful in practice, since a number of observations at different flux densities are usually made upon a single specimen, and the correction may be made once for all to the mass. The quantity $(M+m)(1-1.4c)$ may be called the "effective mass."

At first, a set of corner pieces was made for each set of test pieces of the same material, but this has been found unnecessary. Corner pieces of approximately the same quality and thickness may be used with satisfactory and reliable results, providing the constants of the material are known. The Bureau has now accumulated a sufficient variety of corner pieces so that it is seldom necessary to make a new set, unless unusual precautions are to be taken. When using corner pieces of different material from the test pieces, it is necessary to compute the loss in the entire corner pieces, and then determine an "effective mass" resulting from the lap reducing the flux in the test pieces. Since the thickness of corner pieces may be different from that of the test pieces, it is necessary to consider this, and the flux at the lap may be considered to divide evenly between the two, or in proportion to their thickness. As the results do not differ materially, we assume that in each lapped part the flux density is half the value in the rest of the material.

Let

t = thickness of test pieces,
t_1 = thickness of corner pieces,
w = loss per unit mass in corner pieces,

and other quantities as before. We neglect leakage which is small. c must now be computed by using for M the mass M_1 of test pieces of same material as corner pieces. The loss in the corner pieces, if there were no lap, would be $wm\left(\frac{t}{t_1}\right)^x$. Considering the effect of lap it is $w\ (m-0.7\ cM_1)\left(\frac{t}{t_1}\right)^x = W_c$. The correction to the loss in test pieces due to lap is $0.7c\ (W-W_c)$ and the loss per unit mass is $\frac{W-W_c}{M(1-0.7c)}$. If the corner pieces are of the same thickness as the test pieces, the loss in them becomes $w(m-0.7\ cM_1)$, and this expression will usually give the correction closely enough. Here again the quantity $(m-0.7\ cM_1)$ can be determined once for an entire set of measurements.

If the flux at the lap be assumed to divide between the two in proportion to thickness, we have for the loss in the corner pieces

$$w\left[(m-cM_1)\left(\frac{t}{t_1}\right)^x + cM_1\left(\frac{t}{t+t_1}\right)^x\right]$$

and the correction for lap in the test pieces is

$$c\ (W-W_c)\left[1-\left(\frac{t}{t+t_1}\right)^x\right]$$

The leakage with this arrangement of test material, i. e. the difference between flux at ends of test piece and at middle is usually not greater than 1 per cent. Since the greatest length of a line of induction is 8 per cent larger than the least, there is a probability of the same difference in flux density between the outer and inner sheets. As the average flux density is measured, this can not make an error greater than a small fraction of 1 per cent.[12] Inequality in the four arms has an equally slight effect. Upon removing 1 sheet in 12 from opposite sides, the loss per unit mass was not appreciably altered.

OBSERVATIONS AND RESULTS.

The procedure followed in making a test is as follows. The material is cut into strips of the given dimensions by the use of a sharp machine shear with nearly parallel jaws. The number of strips is determined by the thickness, and for gage No. 29 amounts to 48. The strips are then weighed, bundled and mounted in the solenoids. The effective voltage corresponding to any given flux density and frequency is computed from the following relation:

$$E = 4\, fNn\phi 10^{-8} = \frac{4.44 \times 180 \times 10^{-8}\, Bn\, M}{101.6\rho}$$

where n = frequency, ϕ = total flux, f = form factor of secondary emf., and ρ = density. In the work here reported ρ has been taken as 7.77 grams per cc, but it is proposed hereafter to determine the density for each specimen.

The dynamometer-voltmeter is calibrated for one voltage as determined above, and when taking observations for watt loss, the generator voltage is adjusted until the same deflection is obtained. For other frequencies and flux densities, the resistance in the voltmeter circuit is altered until it is proportional to the product Bn, so that the same deflection is always used. For the lower values of this resistance, the slight correction due to

TABLE I.

SPECIMEN K.

Test Pieces = 1327 Grams. Corner Pieces = 80.2 Grams. c = 0.029. Effective Mass = 1350 Grams.

Cycles	Flux Density	Wattmeter Deflection	Watts	Instrument Losses	Iron Losses	Joules per Cycle	Ergs per Gram per Cycle	Hysteresis	Eddy Currents
60	10000	21.38	4.276	0.079	4.197	0.0699	518	394	124
30	10000	18.87	1.887	.0395	1.848	.0616	456	394	62
60	5000	13.54	1.354	.0395	1.314	.0219	162	130	32
30	5000	12.20	0.610	.0197	0.590	.0197	146	130	16

[12] See M. G. Lloyd, this Bulletin, 5, p. 435; 1909. Reprint No. 108.

the inductance of the instrument is also made. In computing the power supplied to the voltmeter circuit, it may then be remembered that this energy is also proportional to Bn, since the same current is used throughout. A similar series of resistances is usually used in the potential circuit of the wattmeter, so that the power consumed here is also readily computed. The deflection here, however, will increase as the product Bn increases, but will usually remain within working limits of permissible deflections. These limits are so chosen that within them the deflections are proportional to the watts. The multipliers for the potential circuit are so chosen that for one of them the actual watts corresponding to a given deflection, when multiplied by $\frac{N_1}{N_2}=\frac{1000}{180}$, is numerically equal to the deflection. The wattmeter is then direct reading for this range for the values of the watts required, and for other ranges a simple factor, determined by the value of Bn, gives the desired result.

The generator used gives an emf. wave which is sufficiently close to the sinusoidal, and the form factor of the secondary emf. has been determined and found sufficiently near to that assumed through the working ranges of flux density at the frequencies used.

TABLE II.

SPECIMEN H_1.

Test Pieces = 1304 Grams. Corner Pieces of Specimen K = 80.2 Grams. c = 0.029. Effective Mass = 1278 Grams.

Cycles	Flux Density	Wattmeter Deflection	Watts	Instrument Losses	Iron Losses	Joules per Cycle	Corner Pieces	Test Pieces	Ergs per Gram per Cycle
60	10000	21.55	4.310	0.076	4.234	0.0706	0.0028	0.0678	531
30	10000	18.95	1.895	.038	1.857	.0619	.0024	.0595	465
60	5000	13.74	1.374	.038	1.336	.0223	.0009	.0214	168
30	5000	12.34	.617	.019	0.598	.0199	.0008	.0191	150

The frequencies ordinarily used are 60 and 30 cycles, the latter being chosen because it makes the separation of hysteresis and eddy current losses easy to compute. The generator is driven by an electric motor whose field circuit contains a rheostat in the laboratory, permitting adjustment of speed for a definite frequency. The motor is supplied with power from a storage battery so that the speed may be maintained steady. The field circuit of the generator is connected through another rheostat in the laboratory, which permits adjustment of the generator voltage to give the flux density desired. No rheostat is used in the magnetizing circuit. When necessary, a transformer of ample capacity is used to step up or down to the voltage required for the test. The electrical connections are shown in Fig. 3, where for simplicity a single secondary is represented, and may indeed be used in practice. When the generator voltage has been ad-

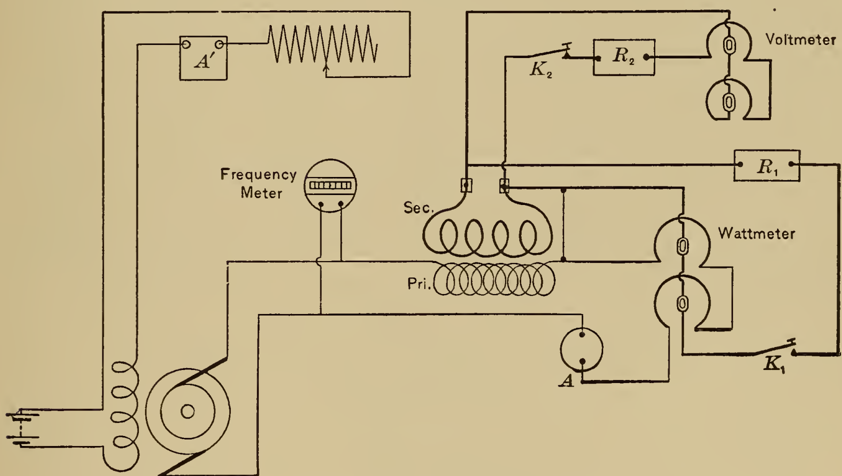

Fig. 3.—Diagram of Connections.

justed to give the proper reading on the voltmeter connected to the secondary circuit, the wattmeter is read. These settings are repeated twice. An adjustment is then made for a different flux density and readings taken as before. When the magnetizing current is desired, an ammeter is included in the magnetizing circuit, and its indications noted. Whenever a change is made from a higher to a lower flux, the current is reduced gradually to the lower value in order to demagnetize the material, and this descending set of observations is usually made at a frequency of 30 cycles. Whenever the magnetizing circuit has been broken, it is closed through a considerable resistance, which is continuously reduced to zero in order to prevent a large first surge and consequent high magnetization, which would require subsequent demagnetization.

Table I gives a specimen set of observations where the corner pieces were of the same material, while in Table II they were of different material.

The total loss is separated into two components, due respectively to hysteresis and eddy currents, as follows, using the Steinmetz equation,

$$W = \eta n B^x + \zeta n^2 B^y$$

where the symbols have the same significance as before, η and ζ being constants of the material. By taking observations at two frequencies, n_1 and n_2, we have

$$\frac{W_1}{n_1} = \eta B^x + \zeta n_1 B^y = a + bn_1$$

$$\frac{W_2}{n_2} = \eta B^x + \zeta n_2 B^y = a + bn_2$$

where a is the hysteresis loss per cycle and bn the eddy current loss per cycle.

$$a = \frac{\frac{W_2}{n_2} n_1 - \frac{W_1}{n_1} n_2}{n_1 - n_2}$$

$$b = \frac{\frac{W_1}{n_1} - \frac{W_2}{n_2}}{n_1 - n_2}$$

If $n_1 = 2\ n_2$ this computation is greatly simplified; for then

$$bn_2 = \frac{W_1}{n_1} - \frac{W_2}{n_2}$$

$$bn_1 = 2bn_2$$

$$a = 2\frac{W_2}{n_2} - \frac{W_1}{n_1} = \frac{W_2}{n_2} - bn_2$$

While the Steinmetz equation, and consequently this separation, is not accurately in accordance with the facts, the errors are very

TABLE III.

Variation of Exponents with Flux Density.

SPECIMEN P_1.

Cycles	Flux Density.	Loss per Cycle	Hysteresis	Eddy Currents	Exponents Hyst.	Exponents E. C.
60	12000	842.7	445.7	397		
30	12000	644.2	445.7	198.5	2.12	2.03
60	10000	576.8	302.8	274		
30	10000	439.8	302.8	137	1.75	1.97
60	8000	381.4	204.8	176.6		
30	8000	293.1	204.8	88.3	1.68	1.83
60	6000	230.9	126.5	104.4		
30	6000	178.7	126.5	52.2	1.53	1.97
60	4000	114.9	67.9	47.0		
30	4000	91.4	67.9	23.5	1.51	2.03
60	2000	35.4	23.8	11.6		
30	2000	29.6	23.8	5.8		

TABLE IV.

Variation of Exponents with Flux Density.

SPECIMEN K.

Cycles	Flux Density	Loss per Cycle	Hysteresis	Eddy Currents	Exponents Hyst.	Exponents E. C.
60	12500	820	638	182		
30	12500	729	638	91	2.16	1.7
60	10000	518	394	124		
30	10000	456	394	62	1.71	1.9
60	7500	313	241	72		
30	7500	277	241	36	1.53	1.9
60	5000	162.2	129.2	33		
30	5000	145.7	129.2	16.5	1.47	2.0
60	2500	54.6	46.6	8.0		
30	2500	50.6	46.6	4.0		

TABLE V.

Variation of Exponents with Flux Density.

SPECIMEN W_1.

Cycles	Flux Density	Loss per Cycle	Hysteresis	Eddy Currents	Exponents	
					Hyst.	E. C.
60	12500	379	309	70		
30	12500	344	309	35	2.04	2.0
60	10000	241	196	45		
30	10000	218.5	196	22.5	1.68	1.9
60	7500	146.8	120.8	26.0		
30	7500	133.8	120.8	13.0	1.60	2.0
60	5000	74.7	63.1	11.6		
30	5000	68.9	63.1	5.8	1.59	2.2
60	2500	23.6	21.0	2.6		
30	2500	22.3	21.0	1.3		

TABLE VI.

Variation of exponents with Flux Density.

SPECIMEN R.

Cycles	Flux Density	Loss per Cycle	Hysteresis	Eddy Currents	Exponents	
					Hyst.	E. C.
60	14000	606	524	82		
30	14000	565	524	41	1.88	1.6
60	12000	456	392	64		
30	12000	424	392	32	1.71	2.1
60	10000	331	287	44		
30	10000	309	287	22	1.68	1.7
60	8000	227	197	30		
30	8000	212	197	15	1.58	2.2
60	6000	141	125	16		
30	6000	133	125	8	1.65	1.6_5
60	4000	72.2	64	8.2		
30	4000	68.1	64	4.1	1.69	1.8
60	2000	22.1	19.7	2.4		
30	2000	20.9	19.7	1.2		

small in thin sheets. The exponents x and y can be determined by observations at different flux densities, and these have been computed for a number of specimens, as shown in Tables III to VII. Where the eddy current loss is small, as in silicon-steel, the values of y are subject to greater error.

While these exponents do not exhibit any definite and constant value, it will be noticed that the hysteresis exponent does not differ much from 1.6 for flux densities between 5000 and 10000 gausses, while for densities exceeding 10000 it is in the neighborhood of 2, with a definite tendency upward. The exponents for eddy current loss vary rather widely from 2 in some instances, but with specimen C in Table VII, where the eddy current loss is greater and the results consequently more accurate, the values come close to 2.

TABLE VII.

Variation of Exponents with Flux Density.

SPECIMEN C.

Cycles	Flux Density	Joules per Cycle	Hysteresis	Eddy Currents	Exponents	
					Hyst.	E. C.
60	13000	.3554	.2872	.0682		
30	13000	.3213	.2872	.0341	1.82	1.96
60	11000	.2611	.2119	.0492		
30	11000	.2365	.2119	.0246	1.59	2.00
60	9000	.1868	.1540	.0328		
30	9000	.1704	.1540	.0164	1.53	1.98
60	7000	.1247	.1047	.0200		
30	7000	.1147	.1047	.0100	1.48	1.93
60	5000	.0741	.0637	.0104		
30	5000	.0689	.0637	.0052	1.43	1.96
60	3000	.0345	.0307	.0038		
30	3000	.0326	.0307	.0019		

In order to test the proportionality between eddy current loss and frequency, some runs were made with a second generator which gave 180 cycles at normal speed and 90 cycles at half speed. The wave form of this generator is not so pure as that of the one used for the lower frequencies, but the loss could be altered by less than 2 per cent at most from this cause, and probably not over 1 per cent. The error would be less at 180 cycles than at 90 cycles from this cause. The measured loss, however, comes a great deal lower at 180 cycles than is computed from the results at 30 and 60

TABLE VIII.

Variation of Eddy Current Loss with Frequency.

SPECIMEN C.

B = 5000 *gausses.* *Thickness* = 0.0422 *cm.*

Cycles	Joules per Cycle	Hysteresis	Eddy Currents observed	Eddy Currents computed	Difference per cent
30	.06844	.06221	.00623		
60	.07467	.06221	.01246		
90	.08074	.06221	.01853	.01869	0.9
180	.09626	.06221	.03405	.03738	9.8

SPECIMEN P.

B = 5000 *gausses.* *Thickness* = 0.0437 *cm.*

Cycles	Joules per Cycle	Hysteresis	Eddy Currents observed	Eddy Currents computed	Difference per cent
30	.02385	.01944	.00441		
60	.02826	.01944	.00882		
90	.03249	.01944	.01305	.01323	1.5
180	.04329	.01944	.02385	.02646	11.

SPECIMEN A_1.

B = 3000 *gausses.* *Thickness* = 0.16 *cm.*

Cycles	Joules per Cycle	Hysteresis	Eddy Currents observed	Eddy Currents computed	Difference per cent
30	.0781	.0510	.0271		
60	.1052	.0510	.0542		
90	.1263	.0510	.0753	.0813	7.4
180	.1765	.0510	.1255	.1626	22.8

cycles. This is illustrated by the experiments exhibited in Table VIII, where the hysteresis and eddy current losses are separated by use of the readings at 30 and 60 cycles. The hysteresis is assumed constant, and the resulting eddy current loss does not increase as rapidly as the frequency. The eddy current loss falls off more rapidly, the thicker the specimen. The effect is greatly accentuated in specimen A_1, which consists of sheets 1.6 mm thick, and was tested by using butt joints at the corners of the apparatus. The results for specimen P are plotted in the curve of Fig. 4. The intercept of this curve upon the vertical axis represents the hysteresis loss. A dotted straight line has been drawn through the points for 30 and 60 cycles. We can not be far wrong in assuming that the curve coincides with this at low frequencies, and consequently

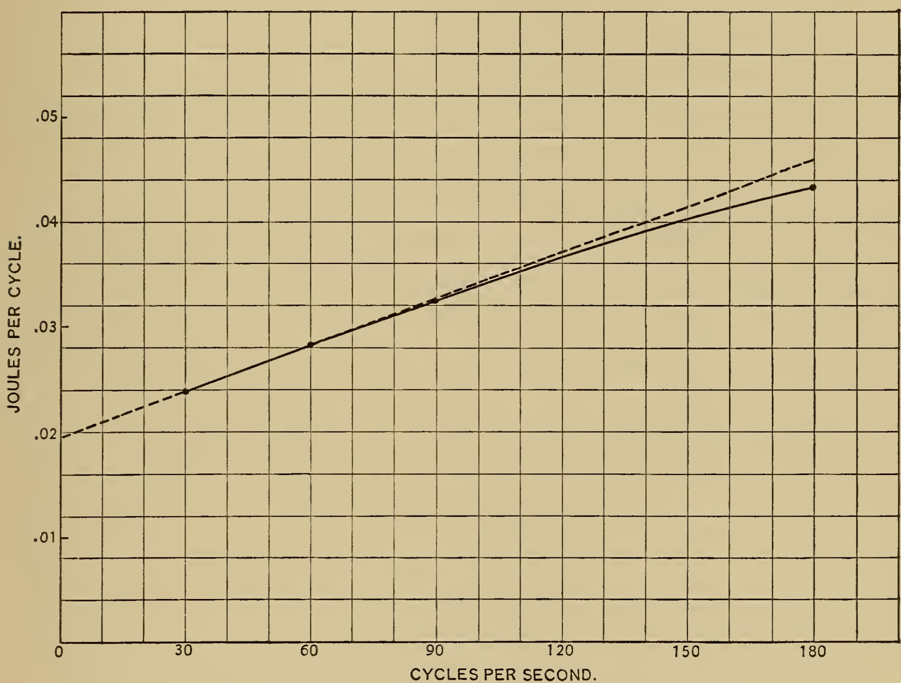

Fig. 4.—Transformer Sheet No. 27 Gage at 5000 gausses.

the method used for the separation of hysteresis and eddy current losses is justified for thin sheets.

The falling off of the eddy current loss at higher frequencies is explained by the consideration that the magnetizing force of the eddy currents reduces the flux in the center and crowds it toward the surfaces of the specimen; the short eddy current paths inclose a smaller flux, while the longest still inclose the same; hence the average emf. of the eddy circuits does not increase as fast as the frequency.

Some experiments were made with thick copper sheets in a solenoid. On account of the low flux densities secured, small

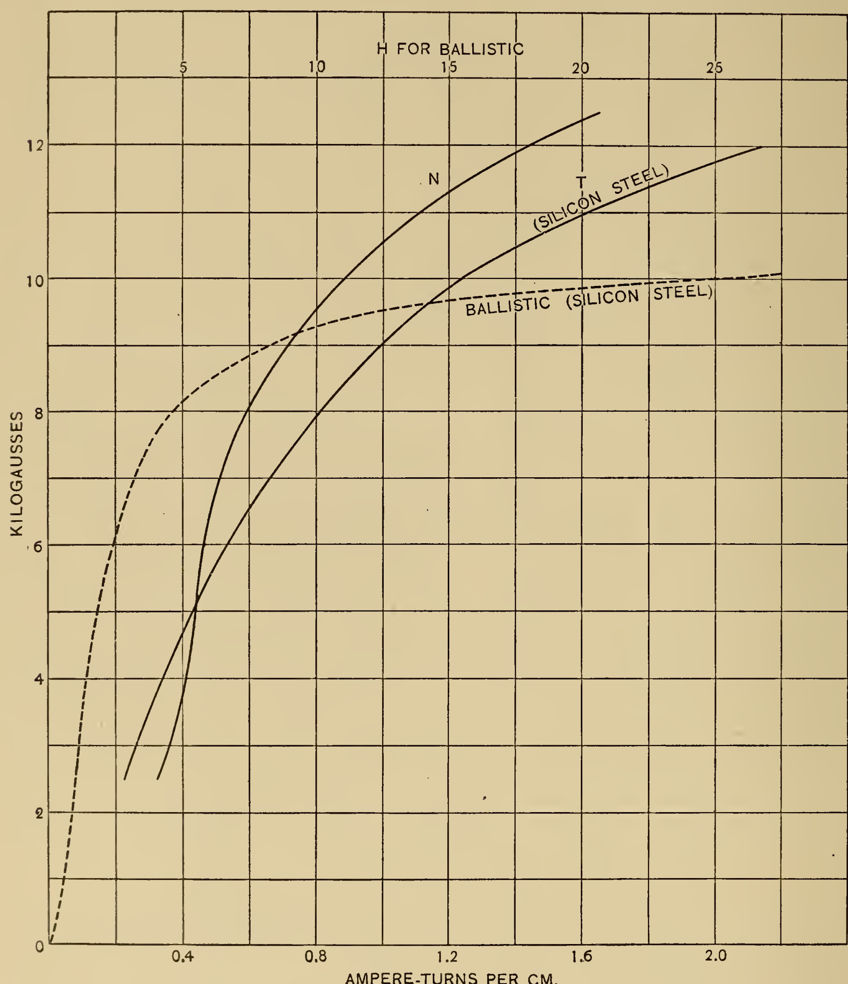

Fig. 5.—Magnetization Curves.

quantities of energy had to be measured, and the accuracy was not high, but the results indicated that the eddy current loss did not increase as rapidly as n^2, but somewhat more rapidly than B^2. The latter result is, however, somewhat doubtful.

By taking readings upon an ammeter in the magnetizing circuit, it is possible to compute the wattless component of magnetizing current, and a curve of such values is plotted in Fig. 5 in relation to the flux density. Such a curve is just as valuable to the designer, or perhaps more valuable, than a magnetization curve obtained by the ballistic method. A similar curve for silicon-steel is plotted in the same figure. A ballistic curve which has been obtained by Dr. C. W. Burrows for a sample of silicon-steel from the same source, but from a different lot, is also shown, but to a different scale. The low magnetizing current required by silicon-steel at low flux densities makes it particularly suitable for current transformers which must have close regulation, as when used with measuring instruments.

TABLE IX.

SPECIMEN N.

Magnetized Parallel to Direction of Rolling.

Cycles	Flux Density	Ergs per Gram per Cycle	Hysteresis	Eddy Currents
60	10000	531	321	210
30	10000	426	321	105
60	7500	318	196	122
30	7500	257	196	61
60	5000	161	105	56
30	5000	133	105	28

Magnetized Normal to Direction of Rolling.

Cycles	Flux Density	Ergs per Gram per Cycle	Hysteresis	Eddy Currents
60	10000	562	352	210
30	10000	457	352	105
60	7500	332	208	124
30	7500	270	208	62
60	5000	167	110	57
30	5000	138.5	110	28.5

That the hysteresis loss is larger when the steel is magnetized normal to the direction of rolling, than when magnetized parallel to the direction of rolling, is shown by Table IX and the curves of Fig. 6. The eddy current loss in the two cases is practically the same, but the hysteresis is 5 to 10 per cent higher for normal magnetization. This specimen is basic open-hearth steel, but is typical of steel from all sources, though the magnitude of the effect is very variable. Table X shows a similar test upon steel from another source.

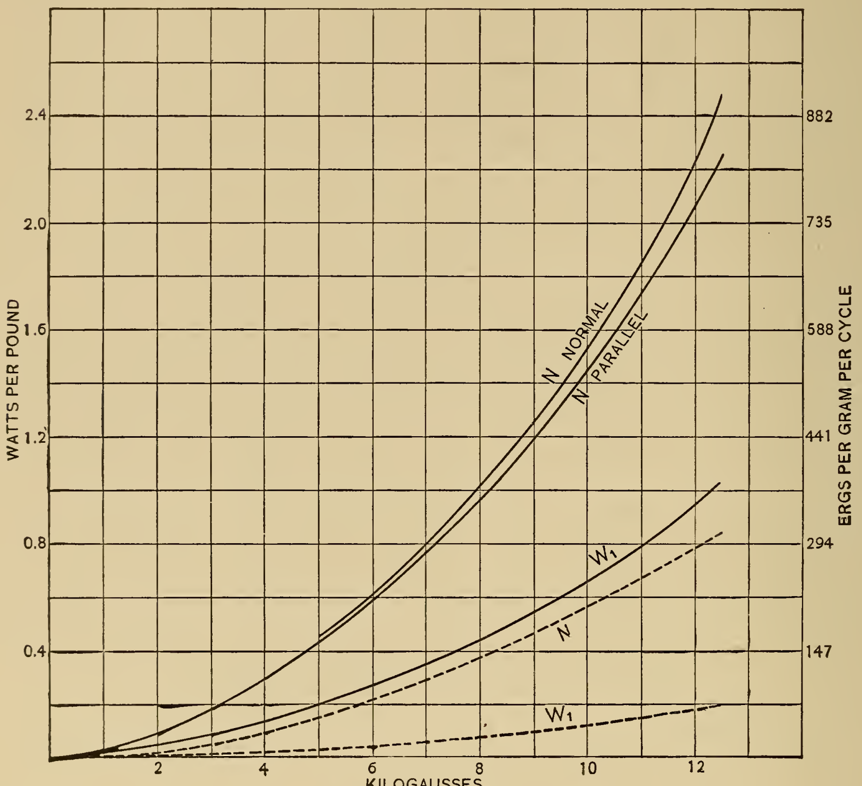

Fig. 6.—Curves showing Relation between Flux Density and Losses when Magnetized parallel and normal to Direction of Rolling. 60 Cycles. Solid Lines represent Total Loss. Dotted Lines show Eddy Current Loss. Difference is due to Hysteresis. N = ordinary steel. W_1 = silicon-steel.

The authors have made tests of sheet iron and steel from a great variety of sources, and have been surprised at the great range in quality of the material in general use by electrical manufacturers, the quality usually having no close relation to the price. Table XI shows some of the results obtained, all the materials having been secured from electrical manufacturers, or from iron mills and dealers supplying the electrical trade. Some foreign samples have been included in the table for comparison. The values of η have been computed from the relation $\frac{W}{n}=\eta B^{1.6}$ where $\frac{W}{n}$ is hysteresis loss in ergs per cc per cycle at 10000 gausses.

TABLE X.

SPECIMEN P_1.

Magnetized Parallel to Direction of Rolling.

Cycles	Flux Density	Ergs per Gram per Cycle	Hysteresis	Eddy Currents
60	10000	576	302	274
30	10000	439	302	137
60	5000	168	96	72
30	5000	132	96	36

Magnetized Normal to Direction of Rolling.

Cycles	Flux Density	Ergs per Gram per Cycle	Hysteresis	Eddy Currents
60	10000	608	336	272
30	10000	472	336	136
60	5000	175	103	72
30	5000	139	103	36

The adjustments can be made and the readings taken so quickly after the circuit is closed, that the specimen becomes heated appreciably only when it is of poor quality or when extremely high frequencies or flux densities are used. The results consequently apply to room temperature. This varies from time to time through several degrees, but it has not been thought necessary to give the temperature in each case. The hysteresis varies only slowly with the temperature, and the eddy current loss, which varies more rapidly, is the smaller part of the total, especially with silicon-steel.

TABLE XI.—Energy Losses due to Alternating

Designation	Thickness	Ergs per Gram per Cycle							
		10000 Gausses				5000 Gausses			
	cm	60 Cycles	30 Cycles	Hysteresis	Eddy Currents at 60~	60 Cycles	30 Cycles	Hysteresis	Eddy Currents at 60~
Unannealed									
A	0.0399	1785	1692	1599	186	608	585	562	46
B	.0326	1290	1223	1156	134	420	402	384	36
C	.0422	1274	1153	1032	242	426	391	356	70
D	.0381	1193	1101	1009	184	401	377	353	48
Annealed									
E	0.0476	971	853	735	236	304	275	246	58
F	.0280	766	716	666	100	247	233.5	220	27
G	.0394	773	668	563	210	247	220	193	54
*H	.0307	558	485	412	146	177.5	158	138.5	39
*H_1	.0277	531	465	399	132	168	150	132	36
J	.0318	543	442	341	202	166.5	139	111.5	55
*K	.0282	518	456	394	124	162	146	130	32
*K_1	.0280	541	479	417	124	170	152	134	36
†L	.0346	565	473	381	184	175	150	125	50
†L_1	.0366	615	516	417	198	192	165	138	54
B_1	.0338	554	454	354	200	173	144.5	116	57
M	.0335	550	461	372	178	173	150	127	46
N	.0340	531	426	321	210	161	133	105	56
N_1	.0312	523	435	347	176	162.5	137.5	112.5	50
P	.0437	518	426	334	184	157	132	107	50
P_1	.0470	576	439	302	274	168	132	96	72
Silicon-Steels									
†Q	0.0361	357	330	303	54	113	105.5	98	15
†Q_1	.0366	390	360	330	60	124	117	110	14
R	.0315	330	309	288	42	104	98.5	93	11
S	.0452	350	314	278	72	108	99	90	18
T	.0338	310	280	250	60	96	87	78	18
U	.0346	312	291	270	42	98	92	86	12
U_1	.0325	322	300	278	44	101	94	87	14
*V	.0310	298.5	275	251.5	47	92	85.5	79	13
*V_1	.0297	303	280	257	46	93	87	81	12
*W	.0305	240	218.5	197	43	74.7	68.5	62.3	12.4
*W_1	.0311	241	218.5	196	45	74.7	68.9	63.1	11.6
X	.0430	265	232.5	200	65	80.8	72.5	64.2	16.6

* German. † English

Magnetization in Various Steels.

x	y	η	Per Cent Silicon	Watts per Pound at 60 Cycles and 10000 Gausses: Eddy Current Loss for Gage No. 29 (See note)	Hysteresis	Total	Designation
							Unannealed
1.51	2.02	0.0049		0.41	4.35	4.76	A
1.59	1.89	.00358		0.44	3.14	3.58	B
1.51	1.79	.00319		0.47	2.81	3.28	C
1.52	1.94	.00312		0.44	2.74	3.18	D
							Annealed
1.58	2.02	0.00227		0.36	2.00	2.36	E
1.60	1.88	.00206		0.44	1.81	2.25	F
1.54	1.96	.00174		0.47	1.53	2.00	G
1.58	1.90	.00127		0.54	1.12	1.66	*H
1.60	1.87	.00123		0.60	1.08	1.68	*H_1
1.62	1.88	.00105	0.0	0.70	0.93	1.63	J
1.61	1.90	.00122		0.54	1.07	1.61	*K
1.62	1.82	.00129	0.4	0.55	1.13	1.68	*K_1
1.61	1.88	.00118		0.535	1.035	1.57	†L
1.60	1.87	.00129		0.515	1.135	1.65	†L_1
1.61	1.81	.00110	0.0	0.61	0.96	1.57	B_1
1.55	1.95	.00115		0.55	1.01	1.56	M
1.62	1.90	.00099		0.63	0.87	1.50	N
1.63	1.81	.00107		0.64	0.94	1.58	N_1
1.64	1.88	.00103	1.3	0.34	0.91	1.25	P
1.66	1.92	.00094	0.7	0.43	0.82	1.25	P_1
							Silicon-Steels
1.63		0.00094	3.1	0.14	0.825	0.965	†Q
1.58		.00102		0.16	0.90	1.06	†Q_1
1.64		.00089	3.4	0.15	0.78	0.93	R
1.63		.00086	3.5	0.12	0.755	0.875	S
1.68		.00077	2.8	0.18	0.68	0.86	T
1.66		.00084		0.12	0.735	0.855	U
1.68		.00086	3.9	0.145	0.755	0.90	U_1
1.68		.00078		0.17	0.685	0.855	*V
1.67		.00080	3.8	0.18	0.70	0.88	*V_1
1.67		.00061	3.4	0.16	0.535	0.695	*W
1.64		.00061		0.16	0.535	0.695	*W_1
1.65		.00062	3.2	0.12	0.545	0.665	X

NOTE.—In order to make a fair comparison the eddy current loss has been computed for a thickness of 0.0357 cm (Gage No. 29), assuming the loss proportional to the square of the thickness.

Specimens *P* and P_1 should perhaps be classed as silicon-steels, although their silicon is not in the proportion which is typical of the alloy. Moreover, they are not put upon the market as an alloy steel, but are sold at about the price of ordinary steel; hence they are classified as such. None of the samples analyzed showed more than the slightest trace of vanadium. Specimen *Q* contained 0.3 of 1 per cent of aluminum. For the chemical analyses we are indebted to Dr. H. C. P. Weber and Mr. J. R. Cain, of this Bureau.

TABLE XII.

Per Cent Increase in Total Loss at 60 Cycles and 10000 Gausses for Different Periods of Aging.

	TIME IN OVEN.					
	100 hrs.	250 hrs.	500 hrs.	750 hrs.	1000 hrs.	2000 hrs.
G	25	58	67	67	68	67
J	1	5	10	12	15	17
K_1		2	6	9	11	
L_1	−2	−1				
P_1		2	3	5		
Q_1	−2	−1				
R	0	0	0			
T		1	2	4		
U_1		0	0	0		
V_1		−1	0	0	1	
W		−2	−1	0	0	
X		0	0	3		

Artificial aging has been practised upon a number of the specimens, and consists in baking in an oven whose temperature is kept between 90° and 100° C. The baking is only interrupted for the purpose of taking observations, which is done after the specimen has cooled to room temperature. The results are shown in Table XII, where the per cent change in total loss is given after various periods of aging. The time given is only approximate. The silicon steels are almost entirely free from aging, but all the other specimens tested aged considerably, except L_1, and the test upon this was not continued for a very long period.

Caution must be exercised in applying the results of such tests, as one may be easily misled by them. Thus the hysteresis and eddy current losses may be differently affected and, moreover, the hysteresis at different flux densities may be differently altered. In the present cases, measurements were also made at 5000 gausses and at 30 cycles. Separation of the losses after aging indicates which component is active. It is found that the hysteresis is nearly always responsible for the increase, although the eddy current loss may be either increased or diminished.

The decrease in eddy current loss may sometimes mask the increase in hysteresis. Thus specimen Q_1 immediately exhibited a decrease of 14 per cent in eddy current loss, amounting to 2 per cent of the total loss. After 250 hours the hysteresis increased 1 per cent at 10000 gausses, and 2 per cent at 5000 gausses, and yet the total loss still shows a decrease. None of the other silicon-steels showed any change in eddy currents, the slight changes in total loss being entirely due to increased hysteresis.

In the specimens G and J the eddy currents at first increased, but in the second thousand hours showed a marked decrease, which was sufficient to balance the steady increase in hysteresis. The result is a nearly stationary total loss. Specimen L_1 showed a slight initial decrease in hysteresis, and a later increase which was masked by decreasing eddy loss.

Specimen P_1 is a good example of the fact that the loss at different flux densities may be differently affected. The hysteresis throughout rose more rapidly at 5000 than at 10000 gausses, the increases amounting after 750 hours to 22 and 16 per cent respectively. This means that the law of variation of hysteresis with flux density has been changed. The exponent of B to which hysteresis is proportional has been diminished from $x = 1.66$ to $x = 1.60$. In the meantime the eddy currents have decreased, so that the increase in total loss at 10000 gausses appears as only 5 per cent. Meanwhile, the total loss at 30 cycles and 5000 gausses has increased 12.5 per cent. It is thus evident that an aging test should be conducted with measurements at the same flux density as that at which the material is to be used; otherwise the results may not apply to working conditions.

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SUMMARY.

The paper contains a discussion of the conditions which should be realized in the measurement of energy losses in sheet iron and steel subjected to alternating magnetization, with a description of a modification of the Epstein method and apparatus, which is believed to better satisfy these conditions and to give an accuracy of 1 per cent with the use of less than 2 kilograms (4.4 pounds) of material.

Results are given showing a wide range in the quality of material in general use, quality having slight connection with price. Several foreign specimens of ordinary steel and silicon-steel are included for comparison with the American product. Silicon-steels contain from 3 to 4 per cent silicon, the quality not depending upon the exact percentage of silicon.

By making measurements at two frequencies, the eddy current and hysteresis losses have been separated and the variation of each with flux density studied. The values of the hysteresis constant and the total loss in watts per pound at 60 cycles and 10000 gausses are tabulated.

The effects of artificial aging are shown to depend upon the flux density selected for test, the hysteresis increasing more for 5000 than for 10000 gausses. Tests should therefore be made at the density to be used. The aging is usually negligible in silicon-steels.

WASHINGTON, January 29, 1909.